Kailu Wang HW 7

1.Proof: Let f: (0,1) 🡪 ℝ defined by f(x)= , we want to show that the function has a limit at 0 and the limit is -. f(x)==. This is a polynomial function which satisfy = f(x0). Then =--=f(0). Hence f has a limit - at 0. ◻

2. Proof: Let f: (0,1) 🡪 ℝ defined by f(x)=. We want to show that the function has a limit at 0 and the limit is -. f(x)= === =. The value of f(0) is independent of the existence of a limit at x=0. Choose δ := ε. Then if x∈ (0,1) with 0< |x| <δ. |f(x)-(- )|= |+= ||= ||= |\*| = ||< |. Since 0<x<1, 18-x>17, and 6>6. Hence, we have proved that f has limit -at 0.

◻

3. Proof: Since f is a monotone, we have two cases. Case1: Suppose that f is increasing. Let L = inf {f(x): a < x ≤ b}. For any ε > 0, since L + ε is not an upper bound of {f(x): a < x ≤ b}, there exists x0 > a such that f(x0) < L + ε. Let δ = x0 − a > 0. Then, for 0 < |x – a| < δ, we have x < x0 and hence L − ε < L ≤ f(x) ≤ f(x0) < L + ε. Thus, f has a limit at a. Now, we let f has a limit at b. f is increasing, then f(b) is an upper bound denote L=sup B. Then L-ε is not an upper bound for B. There is a x0 < b so that L − ε < f(x0) ≤ L. Let δ = b−x0. Then for 0 < |x−b| < δ, we have x0 < x and since f is increasing, then L − ε < f(x0) ≤ L ≤ f(x) < L + ε. Thus, f has a limit at b. Case 2: When f is decreasing, then −f is increasing and hence, −f has limit both at a and at b. This implies that f has a limit both at a and at b.  ◻

4. Proof: Let f,g: D 🡪 ℝ, x0 is an accumulation point of D and f, g has limit at x0. Assume f(x) ≤ g(x) for all x∈ D. Then, there exists a sequence (xn) converges to x0 with x0 ∈ D and xn ≠ x0 and the sequence (f(xn)) and (g(xn)) converges.  ≤, since f(x) ≤ g(x) for all x∈ D and x0 ∈ D. Then ≤.

◻

5. Proof: Assume that f: ℝ 🡪 ℝ is such that f(x+y) = f(x)f(y) for all x,y ∈ ℝ. Let f has a limit at 0. By theorem from class, then we know that . However, as x ->0 x+x ->0. Then we have = ). What’s more, since x-> 0, then -x->0, hence  = )=)=f(0). We have proved that it has Also, there exists an arbitrary real number h as h+0. Then we have f(h+0)=f(h) f(0)=f(h)0=0. Hence for all x∈ ℝ. Let k ≠ 0, 𝑓(𝑥+k)−𝑓(x)=𝑓(𝑥)𝑓(k)−𝑓(x)=𝑓(𝑥)(1−𝑓(k))

==0. Then =1. Hence, we have proved that f has a limit at every point. ◻